The Parkes Sensitivity Calculator

1 Definitions

- k_b is the Boltzmann constant = 1.38×10^{-23} J K^{-1}.
- 1 Jansky (Jy) = 10^{-26} W Hz^{-1} m^{-2}.
- κ is the observing efficiency.
- η_c is the correlator efficiency which is 0.81 for the 2-bit Multibeam correlator, 1.0 for the 8-bit DFBs.
- T_{sys} is the Zenithal frequency-specific system temperature, in Kelvin and includes the 2.75K CMB, typical atmospheric, ground and galactic emission.
- A_e is the antenna effective aperture = $\eta_A A_{geom}$ where η_A is the apperture efficiency and A_{geom} is the geometric area of the antenna in m^2.
- Ω_A is the antenna beam solid angle = $\frac{\pi\theta^2}{4ln2}$ for Gaussian beams with FWHM θ (in steradians).
- n_p is the number of independant polarisations, assumed to be two which are usually averaged.
- $\delta\nu$ is the bandwidth of the observation (in Hz).
- τ is the total (ON[s]+OFF[s]) integration time (in seconds).

2 Introduction

From the radiometer equation, the theoretical minimum detectable temperature is equivalent to the system rms noise temperature:

$$\Delta T_{min}[K] = \frac{\kappa\, T_{sys}}{\eta_c \sqrt{\delta\nu\, \tau}} = \Delta\, T_{rms}$$

The incremental antenna temperature ΔT_A due to a radio source should equal or exceed $\Delta\, T_{min}$ so that $\Delta\, T_A \geq \Delta\, T_{min}$ and so the signal-to-noise ratio becomes $\Delta\, T_A / \Delta\, T_{min}$.

For a point source, of flux density S_ν, we have

$$k_b\, \Delta\, T_A = \frac{1}{2}\, A_e\, S_\nu$$

We can re-cast the above into the 1-σ, minimum detectable flux density for a point source (with $\Delta\, T_A = \Delta\, T_{min} = \Delta\, T_{rms}$):

$$\sigma_S[Jy/beam] = \frac{2\, k_b}{A_e} \Delta\, T_{rms} = \frac{2\, k_b}{A_e} \frac{\kappa\, T_{sys}}{\eta_c \sqrt{n_p\, \Delta\nu\, \tau}}$$

The factor $2\,k_b/A_e$ is also known as the antenna sensitivity, in Jy/K. Values for each receiver can be found in the Online User Guide.

For Parkes receivers ($\nu \sim$ 0.4 - 26 GHz), the Rayleigh-Jeans Approximation holds with $h\nu << kT$ so we can calculate the 1-σ minimal detectable brightness as:

$$\sigma_T[K] = \frac{\lambda^2}{2\,k_b\,\Omega_A}\,\sigma_S$$

3 Determining κ

κ takes into account both the type of observation (beam/position/frequency switching) and the (total) time spent in taking signal and reference spectra.

For total-power observing, the source is always observed, so $\kappa = 1$.

For in-band frequency-switching, half the time is spent on each IF and the noise is increased by $\kappa = \sqrt{2}$.

For position switching or out-of band frequency-switching, $\kappa = 2$ ($\sqrt{2}$ from spending only half the total time on source, and another $\sqrt{2}$ from the subtraction).

For 7-beam MX mode with the 20CM Multibeam, the theoretical factor is $\sqrt{7/6}$ times higher, if all the off-source spectra are used. However, LIVEDATA uses only the off-source spectra in the cycles immediately before and after the on-source cycle, so beams 2-6 are calibrated using an average of two off-source cycles, while beams 1 and 7 use only one. This gives a factor $\kappa = \sqrt{\frac{2+5\times1.5+2}{7}} = 1.28$. For 13-beam MX mode, $\kappa = \sqrt{\frac{2+11\times1.5+2}{7}} = 1.71$.

On-the-fly (OTF) mapping is currently not part of the online sensitivity calculator.

4 Caveats

The online sensitivity calculator does not take into consideration gain instability during observations. It can be shown the total-power radiometer equation which caters for gain fluctuation $\Delta\,G$ is:

$$\Delta\,T_{min} = T_{sys}\sqrt{\frac{1}{\Delta\nu\tau} + \left(\frac{\Delta\,G}{G}\right)^2}$$

The above indicates for gain fluctuations to not significantly degrade the sensitivity, the following must hold:

$$\left(\frac{\Delta\,G}{G}\right) << \frac{1}{\sqrt{\Delta\nu\tau}}$$

Using an example from above, in 7-beam MX mode and correlator configuration mb7_64_2048 and $\tau = 2100$ s, the fractional gain fluctuations need to satisfy

$$\frac{\Delta\,G}{G} << \frac{1}{\sqrt{64\times10^6\times2100}} = 3\times10^{-6}$$

Gain fluctuations typically display a 1/f power spectrum (where f is the observing frequency in Hz). Presently, the effect of 1/f noise is unknown for most if not all Parkes receivers.

5 Examples

With the above formalisation, we present data obtained in various observing modes with both theoretical and observed values of the 1σ rms noise values for line flux. All data were processed with LIVEDATA and GRIDZILLA.

5.1 7-beam MX mode

Data: 2013-03-29_1607-P831_J15472176-6014180.rpf

Receiver: 20cm Multibeam, ν = 1.394 GHz.

Correlator configuration: MBCOR, mb_7_64_2048

Total integration time: 1200 seconds.

Sensitivity calculator estimates: 6.57 mJy/bm or 5.69 mK.

Miriad IMSTAT for line-free region (3000-4500 km/s) gives rms values: 6.29 mJy/bm or 5.10 mK.

5.2 Single-beam position switching mode

Data: 2010-04-03_0039_P706_N_MMS13.rpf

Receiver: 13MM, ν = 23.694 GHz

Correlator configuration: DFB3, sdfb3_256_8192

Total integration time: 960 seconds.

Sensitivity calculator estimates: 34.08 mJy/bm or 20.63 mK.

Miriad IMSTAT for line-free channels (-120 - 50 km/s) gives rms values: 34.23 mJy/bm or 20.69 mK.

5.3 Single-beam frequency swtching mode

Data: 2012-08-10_0112-P798_RCW57_fqsw.rpf

Receiver: 10cm concentric, ν = 3.335 GHz

Correlator configuration: DFB3, sdfb3_8_8192_fqsw

Total integration time: 900 seconds.

Sensitivity calculator estimates: 44.80 mJy/bm or 34.48 mK.

Miriad IMSTAT for line-free channels (-20 - 20 km/s) gives rms values 173 mJy/bm or 146 mK.

The gross difference in the observed and theoretical values can be understood as the source in question (RCW57) lies in the Galactic Plane. The recorded T_{sys} at the position of RCW57 is $\sim$ 128K, which is 93K more than the Zenithal T_{sys} of the 10cm receiver (= 35K). If we add 93K to the online sensitivity calculator, we get estimates of 163.84 Jy/bm or 126.11 mK, inline with the theoretical estimates.

6 References

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